Hypothesis Testing

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Outlines

- Introduction
- Hypothesis
- Hypothesis Testing
- Calculation

Expected outcomes

- Understand the concept of hypothesis testing as one of inference methods
- Understand the concepts of hypothesis and hypothesis testing
- Apply the concepts and steps in hypothesis testing to calculate relevant values and make decision

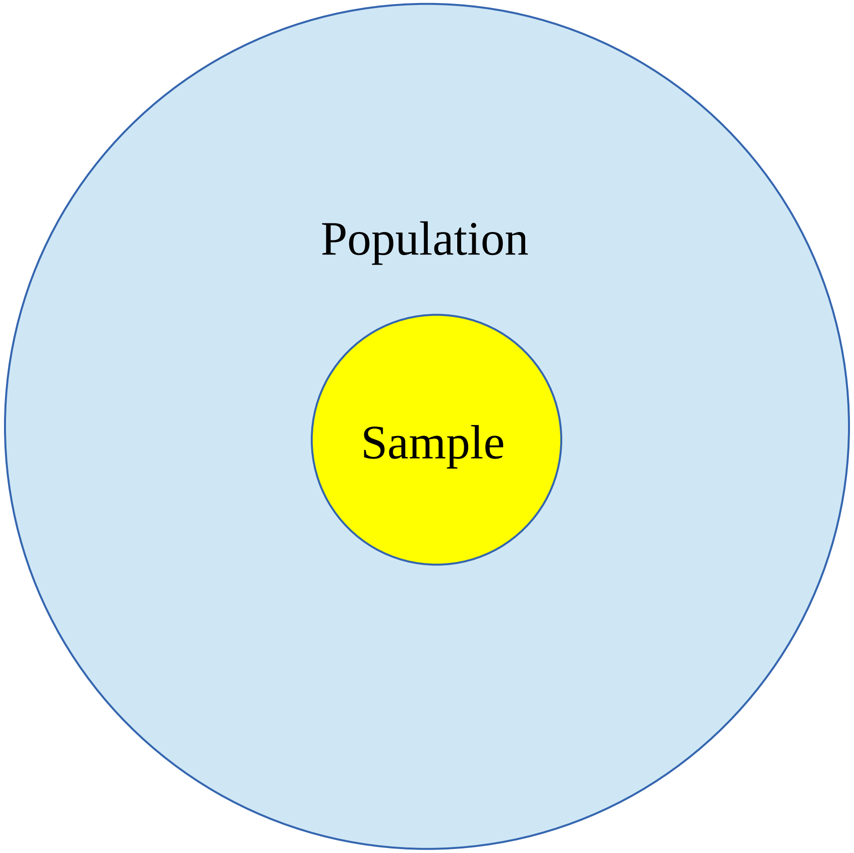
Introduction

Overview

- Statistics is a field of study dealing with (Daniel, 1995):
  1. Collection, organization, summarization and analysis of data.
  2. Making <u>inference/conclusion</u> about population data from sample data.

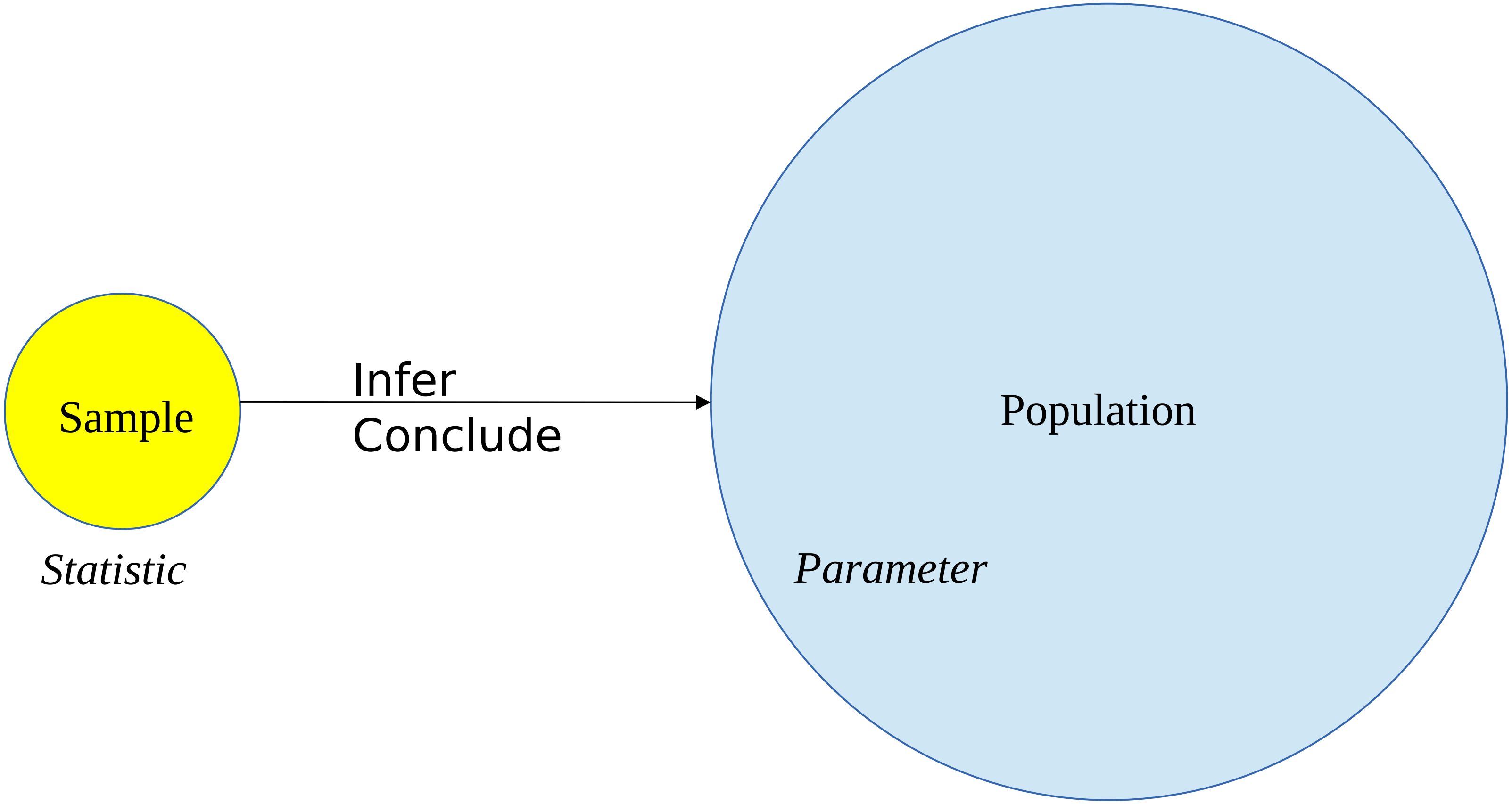
Overview

- Population *vs* sample

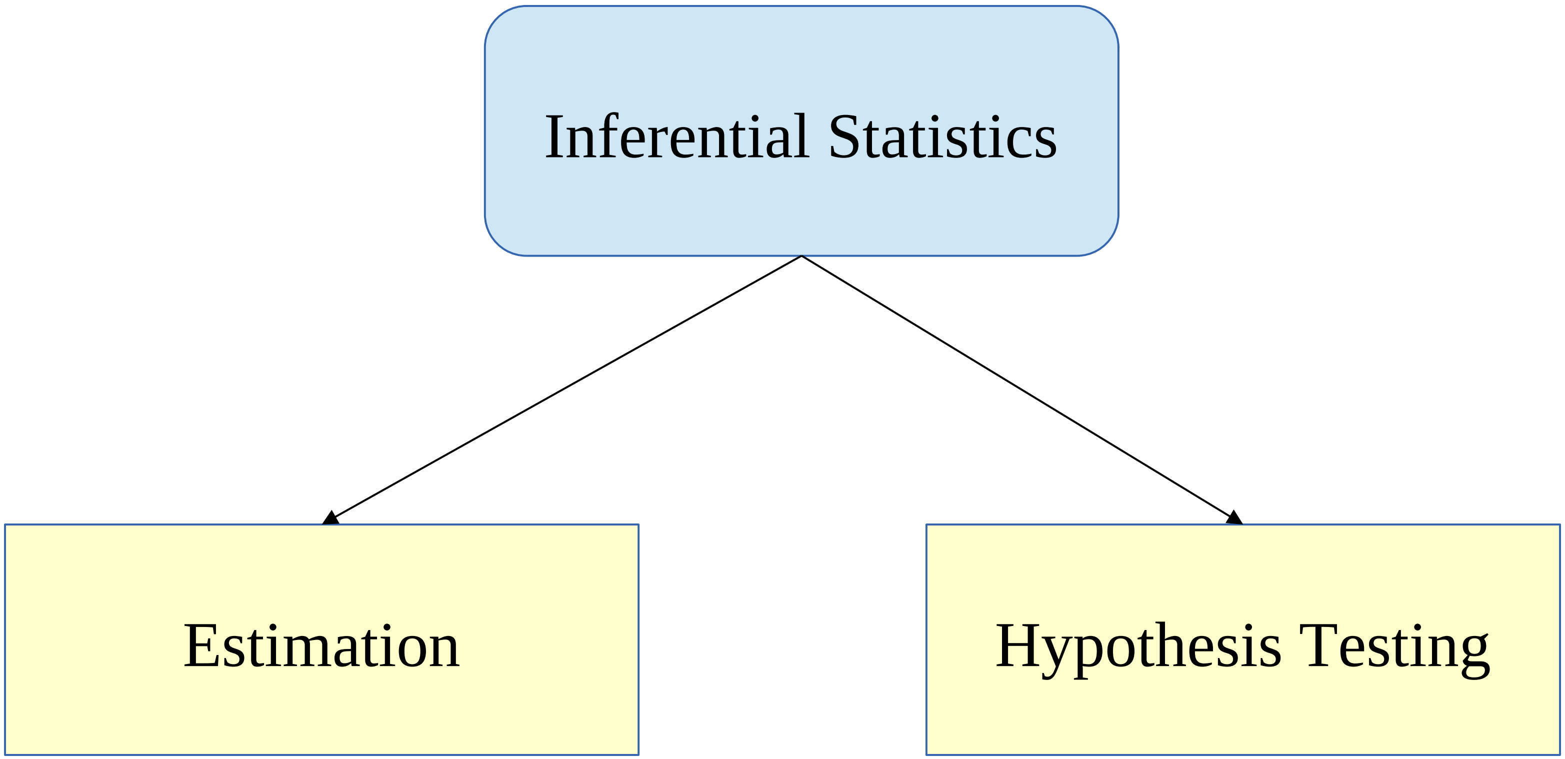


Overview

- Inference:



Overview



Hypothesis

Types of Hypothesis

- Research hypothesis "is the conjecture or supposition that motivates the research"
- Statistical hypothesis is the hypothesis that is stated in a way that is possible to evaluate by appropriate statistical analysis

Types of Statistical Hypothesis

- Null hypothesis (H_0) – It states a hypothesis of no difference/agreement with the population of interest
- Alternative hypothesis (H_A) – The inverse of H_0. It states a hypothesis of disagreement with the population of interest. In most situations, it reflect the proposition in the research hypothesis

Statistical Test & Hypothesis

- **Neyman-Pearson paradigm**
  1. Acceptance region, the subset of the test statistic values for which H_0 is accepted
  2. Rejection region or critical region, the complement of the acceptance region, for which H_0 is rejected and H_A is accepted

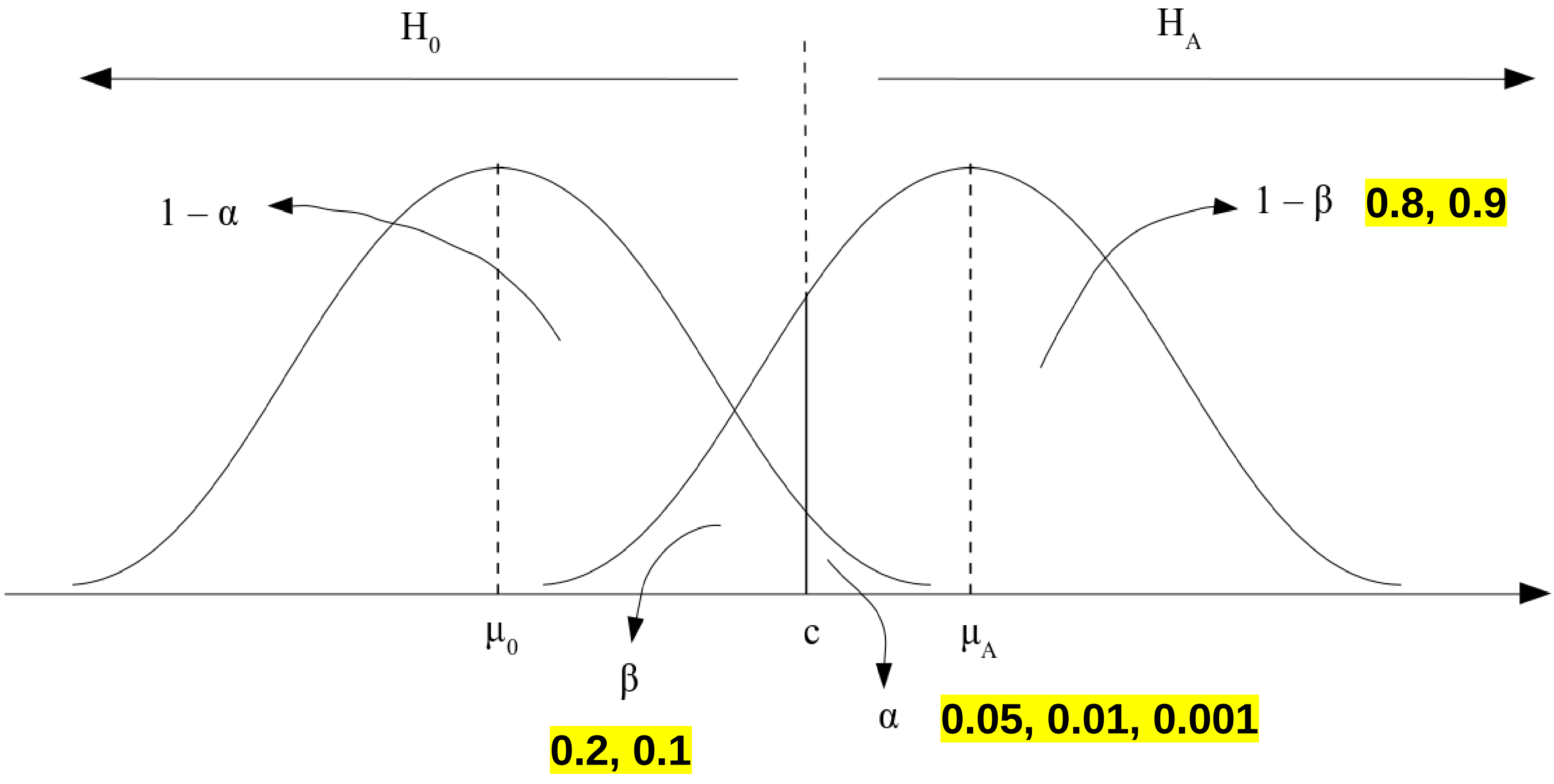
Confidence Interval & Hypothesis Testing

- Acceptance region, region covered by a confidence interval, H_0 is accepted
- Rejection region, region outside the confidence interval at both tails for a two-tailed test, or at either tail for a one-tailed test, H_0 is rejected and H_A is accepted

Statistical Test vs Hypothesis

		Hypothesis state **(Truth)**	
		True H_A False H_0 (Difference +)	True H_0 False H_A (Difference –)
Statistical test result	Significant (Test +)	True Positive $1 - \beta$ Sensitivity/**Power**	False Positive α **Type I Error**
	No significant (Test –)	False Negative β **Type II Error**	True Negative $1 - \alpha$ Specificity

Statistical Test vs Hypothesis



Test Statistic

- Statistic obtained from our calculation using appropriate formula
- General formula for hypothesis testing

$$\text{test statistic } W(\boldsymbol{X}) = \frac{\text{relevant statistic} - \text{hypothesized parameter}}{\text{standard error of relevant statistic}}$$

- Compared against a preset value OR
- Converted into probability value called *P*-value

Significance level, α

- The probability of rejecting a true null hypothesis
- Preset value of acceptable significance level (0.05, 0.01, 0.001), cutoff *c*
- Acceptable level to commit type I error
- Compared to a test's *P*-value OR converted to test statistic value

P-value

- The probability of getting a test statistic as extreme as the one that calculated
- Very low *P*-value means that the probability of getting a value similar or better is very unlikely
- When *P*-value < α → significant result, it would not happen easily by chance

Hypothesis Testing

Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

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- Nature of data: numerical/categorical
- Present basic information: *n*, mean, variance/standard deviation

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- Distribution: normal/not normal
- Random samples
- Independence of samples: independent/paired
- Population variance/standard deviation: known/not known

Steps

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- H_0 and H_A can be stated in three ways based on H_A

1) Difference (two-tailed test)

Population mean SBP is not 120mmHg,

$$H_0: \mu = 120$$

$$H_A: \mu_A \neq 120$$

Steps

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8. Conclusion

- H_0 and H_A can be stated in three ways based on H_A

2) More (one-tailed test)

Population mean SBP is more than 120mmHg,

$$H_0: \mu_A \leq 120$$

$$H_A: \mu_A > 120$$

Steps

1. Data description
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3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

- H_0 and H_A can be stated in three ways based on H_A

3) Less (one-tailed test)

Population mean SBP is less 120mmHg,

$$H_0: \mu_A \geq 120$$

$$H_A: \mu_A < 120$$

Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
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7. Statistical decision
8. Conclusion

- Depending on the data and assumptions, state an appropriate statistic to use
- For example, for normally distributed data, when population variance is unknown → t statistic instead of z

Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

- Set the significance level, usually $\alpha = 0.05$
- Determine the cut-off value of the test statistic at the significance level
- Specify rejection/acceptance region
- One-tailed or two-tailed test based on the hypothesis statement → common two-tailed
- If test statistic is outside the acceptance region/within rejection region → significant test, reject H_0, accept H_A
- If test statistic is within the acceptance region → test not significant, accept H_0, reject H_A

Steps

1. Data description
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- Based on the chosen test statistic

Steps

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7. Statistical decision
8. Conclusion

- Reject/accept H_0
- Significant/not significant given the preset significance level, α
- State the *P*-value
- For the two-tailed *P*-value, multiply the probability obtained from the probability distribution by two

Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

- Conclude in relation to the research hypothesis

Calculation

One Population Mean

- Test statistic for one population mean

$$\text{test statistic} = \frac{\text{sample mean} - \text{hypothesized mean}}{\text{standard error of mean}}$$

Standard normal distribution, *z*

- The test statistic for one population mean

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{(n)}}}$$

given H_0, is distributed as z

- population variance σ^2 is known
- data normally distributed

Calculate

- A study was done in a sample of 30 adults to investigate the systolic blood pressure (SBP) in a population.
- It was hypothesized the mean SBP is 120mmHg.
- The data was normally distributed, with the mean SBP of 125mmHg and standard deviation s of 15mmHg.
- The sample *s* is treated as the population σ.
- Following the general hypothesis testing steps, based on the sample data, decide on the given hypothesis.

Calculate

H_A: $\mu \neq 120$

For two-tailed, depending on value of sample mean (more or less than population mean), chose to calculate based on right/left tail

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{(n)}}}$$

$$n = 30$$
$$\bar{x} = 125$$
$$s = 15$$
$$\mu_0 = 120$$

$$P(SBP > 125)$$
$$= P(Z > \frac{125 - 120}{15/\sqrt{30}})$$
$$= P(Z > 1.826)$$
$$= 1 - P(Z < 1.826)$$
$$= 1 - 0.966$$
$$= 0.034$$ Multiply by 2 for two tailed

Also refer to **hypothesis.R**

$P\text{-value} = 0.034 \times 2 = 0.068$ > 0.05, we do not reject Null Hx

Calculate

H_A: $\mu \neq 120$

For two-tailed, at significance level = 0.05, critical values between -1.96 to 1.96, as z

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{(n)}}}$$

$$n = 30$$
$$\bar{x} = 125$$
$$s = 15$$
$$\mu_0 = 120$$

$$z = \frac{125 - 120}{15/\sqrt{30}} = 1.826$$

Therefore, 1.826 in within the range, we do not reject Null Hx

Calculate

H_A: $\mu > 120$

$$P(SBP > 125)$$
$$= P\left(Z > \frac{125-120}{15/\sqrt{30}}\right)$$
$$= P(Z > 1.826)$$
$$= 1 - P(Z < 1.826)$$
$$= 1 - 0.966$$
$$= 0.034$$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{(n)}}}$$

$n = 30$
$\bar{x} = 125$
$s = 15$
$\mu_0 = 120$

Also refer to **hypothesis.R**

P-value $= 0.034$ < 0.05, we reject Null Hx

Calculate

H_A: $\mu > 120$

For one-tailed, at significance level = 0.05, critical value at right tail is 1.65, as z

$$z=\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{(n)}}}$$

$$n=30$$
$$\bar{x}=125$$
$$s=15$$
$$\mu_0=120$$

$$z=\frac{125-120}{15/\sqrt{30}}=1.826$$

Therefore, 1.826 > 1.65, we reject Null Hx

Calculate

H_A: μ < 120

$$P(SBP < 125)$$
$$= P\left(Z < \frac{125-120}{15/\sqrt{30}}\right)$$
$$= P(Z < 1.826)$$
$$= 0.966$$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{(n)}}}$$

$$n = 30$$
$$\bar{x} = 125$$
$$s = 15$$
$$\mu_0 = 120$$

Also refer to **hypothesis.R**

P-value $= 0.966$ > 0.05, we do not reject Null Hx

Calculate

H_A: $\mu < 120$

For one-tailed, at significance level = 0.05, critical value at left tail is -1.65, as z

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{(n)}}}$$

$$n = 30$$
$$\bar{x} = 125$$
$$s = 15$$
$$\mu_0 = 120$$

$$z = \frac{125 - 120}{15/\sqrt{30}} = 1.826$$

Therefore, 1.826 > -1.65, as we need z to be less than -1.65 (left tail) we do reject Null Hx

One Population Proportion

- Test statistic for one population proportion

$$\text{test statistic} = \frac{\text{sample proportion} - \text{hypothesized proportion}}{\text{standard error of proportion}}$$

Standard normal distribution, *z*

- The test statistic for one population proportion

$$z = \frac{\hat{p} - p}{\sqrt{\left(\frac{p(1-p)}{n}\right)}}$$

given H_0, is distributed as *z* (provided *np* and *n*(1 – *p*) > 5)

Calculate

- A study was done in a sample of 60 adults to investigate the proportion of diabetics in a population.
- It was hypothesized the proportion of diabetics in the population is 0.3.
- Based on the data, 25 participants were found to be diabetics.
- Following the general hypothesis testing steps, based on the sample data, decide on the given hypothesis.

Calculate

H_A: $p \neq 0.3$

Complete the calculation. Also refer to **hypothesis.R**

Calculate

H_A: $p > 0.3$

Complete the calculation. Also refer to **hypothesis.R**

Calculate

$H_A: p < 0.3$

Complete the calculation. Also refer to **hypothesis.R**